

Differentiation problems

1. Differentiate:

a) $x^4 + 4x^4 - 5x^3 + x + 1$

b) $(x^2 + 3x + 2)^{10}$

c) $[(x^3 + 2x + 1)^6 + (x^5 + x^3 + 2)^5]^{10}$

d) $(3x^3 + 5x + 2)(7x^8 + 5x + 5)$

e) $(5x^7 + 3)/(8x^9 - 3x - 1)$

f) $(x^3 + 1)\sqrt{(x)}$

2. Derive the quotient rule without using the implicit differentiation

3. x and t satisfy the equation $x^7 + x^3 + 3t^4 + 2t + 1 = 0$ find a formula for dx/dt in terms of x and t

4. Use implicit differentiation to derive the formula for $d(x^{p/q})/dx$ where p and q are integers.

5. The area of a disc of radius r is given by the formula $A(r) = \pi r^2$
Does the derivative $A'(r)$ remind you of anything? Can you see any geometric meaning of it?

6. The volume of a 3-dimensional ball of radius r is given by the formula $V(r) = 4\pi r^3/3$

Does the derivative $V'(r)$ remind you of anything? Can you see any geometric meaning of it?

7. Derive a formula for $(f(x)g(x)h(x))'$

For the real enthusiasts:

8. Try to generalize for product of more than 3 functions.

9. Read "Calculating areas" on that single sheet handout with Fermat's picture.

10. Read the handouts "Induction and mathematical induction" and "Recursion" and try to understand how to derive the formulas for $1 + 2 + 3 + \dots + n$, $1 + 4 + 9 + \dots + n^2$, ..., $1 + 2^k + 3^3 + \dots + n^k$